AD-A055 243   WEIBULL (WALODDI) LAUSANNE (SWITZERLAND)   F/G 13/13
THE PROBABILITY OF FAILURE OF A SYSTEM SUBJECTED TO THE JOINT E--ETC(U)
SEP 77   W WEIBULL   F44620-73-C-0066
UNCLASSIFIED   AFML-TR-77-169   NL

| OF |
AD
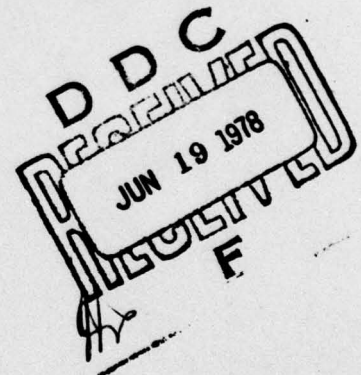A055243

END
DATE
FILMED
7 -78
DDC

AD A055243

AFML-TR-77-169

THE PROBABILITY OF FAILURE OF A SYSTEM SUBJECTED TO THE JOINT
EFFECT OF CYCLIC LOADING AND RANDOMLY DISTRIBUTED DISCRETE
LOAD PEAKS

Avenue d'Albigny, 9 bis
74000 Annecy
France

September 1977

TECHNICAL REPORT AFML-TR-77-169
Final Report for Period 1 August 1973 - 1 October 1973
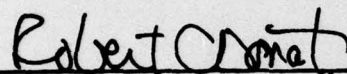
Approved for public release; distribution unlimited

78 06 09 062

NOTICE

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.
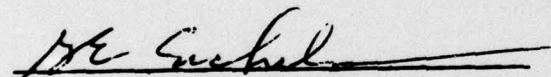
This report has been reviewed by the Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

ROBERT C. DONAT
Metals Behavior Branch
Metals and Ceramics Division

FOR THE COMMANDER

GAIL E. EICHELMAN
Acting Chief
Metals Behavior Branch
Metals and Ceramics Division

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFML-TR-77-169	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) The Probability of Failure of a System Subjected to the Joint Effect of Cyclic Loading and Randomly Distributed Discrete Load Peaks		5. TYPE OF REPORT & PERIOD COVERED Final Report, 1 Aug — 1 Oct 73
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Waloddi Weibull		8. CONTRACT OR GRANT NUMBER(s) F44620-73-C-0066
9. PERFORMING ORGANIZATION NAME AND ADDRESS AFML/LLN Wright-Patterson AFB, OH 45433		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 7351-06 62102F 687351
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE September 1977
		13. NUMBER OF PAGES 12
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

structural failure probability
cumulative fatigue damage
fatigue failure distribution
structural reliability

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

If a specimen is subjected to some kind of cyclic loading with a maximum load level S, which is larger than the fatigue limit of the specimen, then the strength of the specimen will gradually decrease until its residual strength R reaches the value S, when static failure occurs.

This deterioration process may, as indicated, be described graphically by an R-S-N-diagram or analytically by a set of parameter functions of S and N.

If now a discrete load peak of known level L is imposed upon the specimen after N fatigue cycles, then the probability of failure may be directly

DD FORM 1473 EDITION OF 1 NOV 65 IS OBSOLETE

373 000

read from the diagram or computed by use of the parameter functions.

A generalization of the R-S-N-diagram is proposed in order to make it applicable to the case, when the discrete load peaks are replaced by sequences of different cyclic loadings.

By use of this diagram it has been proved that Miner's measure of cumulative fatigue damage $M = \Sigma n_i/N_i$ depends on the order in which the different sequences are applied, a defect which has been repeatedly verified by experiment.

Sum over n sub i / N sub i

FOREWORD

The research work reported herein was conducted by Prof.
Dr. Waloddi Weibull, Avenue d'Albigny, 9 bis, 74000 Annecy,
France under USAF Contract No. F44620-73-C-0066. This con-
tract, which was initiated under Project No. 7351, "Metallic
Materials", Task 735106, "Behavior of Metals", was administer-
ed by the European Office of Aerospace Research. The work
was monitored by the Metals and Ceramics Division, Air Force
Materials Laboratory, Air Force Systems Command, Wright-Pat-
terson Air Force Base, Ohio, under the direction of Mr. W. J.
Trapp, AFML/LL.

This report covers work conducted during the period 1 Au-
gust 1973 to 1 October 1973. The manuscript was submitted by
the author for publication in November 1973.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

1. INTRODUCTION

Consider a specimen subjected to some kind of cyclic loading, random or of constant amplitude, with a maximum load level S. If now $S > S_o =$ the fatigue limit of the specimen, then the specimen will, by definition, fail after a finite number N of cycles. This will happen, when its gradually decreasing residual strength R has reached the value of the applied maximum load level S. Thus, a fatigue failure is, in fact, always in principle a static failure.

It should be noted that S and $R =$ the residual strength of an individual specimen, are real quantities, whereas the residual strength R of a specified type of specimens is a random variable, which has to be defined by a distribution function $F(r) = \text{Prob}(R \leq r)$, such that $F(r_u) = 0$, that is, r_u is the lower limit of the distribution. The function $F(r)$ contains parameters, one of them being r_u, which will gradually change during the fatigue process, thus being functions of S and N.

If now a discrete load peak of level L is imposed upon the specimen at the time N, then nothing will happen, if $L < r_u$, whereas failure may occur, if $L > r_u$. The probability of this failure is given by $P = F(L)$. Thus it is obvious that the fundamental problem of estimating the probability of failure due to the joint effect of a specified cyclic loading and discrete load peaks consists in a determination of the R-S-N-diagram, which provides the relationship between the applied cyclic loading with the maximum load level S, the gradually decreasing residual strength R of the specimen, and the number of cycles N actually imposed upon the specimen.

2. THE R-S-N-DIAGRAM

2.1 Graphical Representations

The random variable R is specified by its distribution function $F(r)$, as indicated in Figure 1. This function provides all information about R. For the purpose of graphical representations of the diagram it will be more convenient to use the percentiles r_p defined by

$$P = F(r_p) \quad \text{or} \quad r_p = F^{-1}(P) \tag{1}$$

and let the R-S-N-diagram be represented by a family of curves of the percentiles

$$r_p = f_p(S, N)$$

1

for a properly selected set of percentage points P, for instance,
P = 0, 10, 50, 90, 99%, as indicated in Figure 2. It is obvious that
this diagram depends on the type of cyclic loading in question, each
of which requires its proper diagram. The distribution of the fatigue
life of the specimen is also provided by the family of the percentiles
N_p, where N_p denotes the number of cycles at which P% of a set of speci-
mens have failed after having been subjected to the same cyclic loading
with the maximum load level S.

The shape of the r_p-curves has to be experimentally determined, as
will be demonstrated in the following by an example. An important
property of them is already given in a report by D. Broek: (1). It
is summarized as follows: "Specimens of 2024-T3 and 7075-T6 aluminum
alloy sheet containing fine saw cuts appear to have the same residual
strength as specimens with fatigue cracks of the same length. The
stress level at which the fatigue cracks were grown does not affect
the residual strength properties." It may thus be concluded that, at
least for this type of specimens, the residual strength is a unique
function of the crack length, which implies that these curves are com-
posed of two parts, the first one corresponding to the initiation and
the second one to the propagation of the crack. During the first period
the residual strength seems to be practically constant, whereas it
will, during the second period, decrease rapidly with the growth of the
crack.

Another conclusion of great importance for the numerical evaluation
of the test data, on which the diagram will be based, is, as indicated
in Figure 2, that the fatigue procedure has to be divided into four
periods:

a) $(0 - N_a)$: None of the specimens contains a fatigue crack.
    The residual strength is practically constant. The distribution
    of R is identical with that of R_o, that is, the virgin strength
    of the specimens.

b) $(N_a - N_b)$: Some of the specimens will contain fatigue cracks.
    During this period the distribution of R is a two-component
    one and has to be specified by two different functions.

c) $(N_b - N_c)$: All specimens will contain fatigue cracks and
    the distribution of R may be specified by a single function.

d) $(N_c - N_d)$: Some of the specimens may have failed. The distri-
    bution of R will be truncated.

2

If now an R-S-N-diagram has been set up, then the probability of failure at the occurrence of a load peak is easily found. For instance, as indicated in Figure 2, a load peak L_1 occurring at the time N_1 will produce a failure with the probability 90%, as may be directly read from the diagram.

2.2 Analytical Representations

For more precise evaluations it is preferable to represent the diagram by a set of parameter functions, as indicated below.

Let us assume the distribution function

$$P = F(r) = 1-e^{-((r-r_u)/r_o)^m} \quad \text{for} \quad r \geq r_u \quad (3)$$

$$= 0 \quad \text{for} \quad r < r_u$$

where m, r_o, r_u are the shape, scale, and location parameters, respectively. It is obvious that r_u is identical with r_p for $P=0$.

The parameters now have to be estimated for any given cyclic loading from measured values of the residual strength of a sufficient number of specimens at several different times N.

For the first and the third period of the fatigue procedure, mentioned above, it will be possible to represent the R-S-N-relationship by a set of three functions of the parameters

$$m = f_m(S,N)$$

$$r_o = f_o(S,N) \quad (4)$$

$$r_u = f_u(S,N)$$

It is believed that these three sets of paramter functions may be used also to specify the partial distributions of the second and fourth periods.

The necessary procedures for setting up such a diagram will now, as an orientation, be illustrated by a test program.

3

3. EXPERIMENTAL DETERMINATION OF AN R-S-N-DIAGRAM

3.1 The Specimens

One hundred aluminum alloy sheet specimens, containing a central, drilled hole, of appropriate dimensions with regard to available testing machines, are fabricated at the same time. Five groups, Nrs. 1-5, each of nine specimens, are taken out at random.

3.2 The Virgin Tensile Strength R_o

The tensile strength of the specimens of Group Nr. 1 are measured and plotted in the diagram as the first points of the r_p-curves, corresponding to the probabilities P = 10, 20, ..., 90%. They will be denoted by R_{o1}, R_{o2},...,R_{o9}.

3.3 Specification of the Cyclic Loading

For this first illustration of the test procedure the most simple type of cyclic loading will be used, consisting in axial load cycles of constant amplitude with the lower load level $S_{min} = 0$ and the upper load level $S_{max} = 0.75.R_{o1}$. It is believed that this cyclic loading will satisfy the requirement that all specimens subjected to it will fail within a reasonable number of cycles.

3.4 The Fatigue Life N_o

The specimens of Group Nr. 2 are subjected to the above-mentioned cyclic loading until all of the specimens have failed. The fatigue lives, denoted by N_{o1},...N_{09}, are plotted in the diagram.

3.5 The Residual Strength R_1

The specimens of Group Nr. 3 are subjected to the cyclic loading during a time $N_1 = 0.5.N_{o1}$, after which the fatigue procedure is discontinued, and the residual strength of the specimens are measured. It is believed that none of the specimens will fail in fatigue. The strengths, denoted by R_{11}, R_{12},...R_{19} are plotted in the diagram.

3.6 The Residual Strength R_2

The specimens of Group Nr. 4 are subjected to the cyclic loading during the time $N_2 = N_{o1}$, after which the fatigue procedure is discontinued, and the residual strength of the unfailed specimens, denoted by R_{21}, are measured and plotted in the diagram.

4

3.7 The Residual Strength R_3

The specimens of Group Nr. 5 are subjected to the cyclic loading during a time $N_3 = N_{05}$. About fifty percent of the specimens are expected to fail in fatigue. The residual strength of the remaining specimens, denoted by R_{31}, are measured and plotted in the diagram.

If now nine curves, each passing through the five points of the same order number are drawn, the R-S-N-diagram is completed. The shape of this diagram will decide the continued use of the remaining 55 specimens, in order to obtain the best possible estimates of the parameter functions (4).

4. A GENERALIZED R-S-N-DIAGRAM

The R-S-N-diagram was initially designed for the purpose of predicting the life of a system in an operational environment of a specified cyclic load and superimposed, randomly distributed, discrete load peaks.

It can, however, be adapted for more general purposes, viz., for environments where the discrete load peaks are replaced by sequences of different cyclic loads. We thus arrive at the problem of predicting life of systems which are subjected to several different cyclic loadings, which. follow after each other in a prescribed order during prescribed periods.

This problem will be solved by setting up, for each involved, specified cyclic loading, its R-S-N-diagram and combining these diagrams.

The procedure will be illustrated by the simple case of two different cyclic loadings with the maximum load levels S_1 and S_2, as indicated in Figure 3. For each of them the family of r_p-curves are supposed to be known, but for the further discussions only one of each are needed, r_{1p} and r_{2p}, corresponding to the same but so far unspecified percentage point P. These two curves start at the same point R_{op} but end at different fatigue lives, N_{1P} and N_{2p}.

Let us now examine with reference to Figure 3 two different programs.

Program 1: Load S_1 is applied during $n_1 = 0.5.N_{1P}$ cycles, resulting in a reduced residual strength R_1 after which Load S_2 is applied until failure occurs with a probability of P%. The residual strength R_1, which was obtained by Load S_1, after n_1 cycles, could have been obtained by Load S_2 after n_2 cycles.

5

According to the finding of Broek, the residual strength properties are not affected by the stress levels at which the cracks are grown. Hence it can be concluded that, if Load S_2 is applied, failure will occur after $(N_{2P} - n_2)$ cycles.

If now this program, that is, Load S_1, is applied during n_1 cycles and Load S_2 during $(N_{2P} - n_2)$ cycles, is imposed upon a number of specimens and Diagram 3 corresponds to P=0, then it follows that none of the specimens will fail. If, on the other hand, Diagram 3 corresponds to P = 50%, then fifty percent of them will fail.

Program 2. Load nr.S_2 is applied during $n_3 = 0.5 . N_{2P}$ cycles after which Load nr.S_1 is applied during $(N_{1P} - n_4)$ cycles, when the specified failure will occur.

5. CHECKING THE MINER'S RULE OF CUMULATIVE FATIGUE DAMAGE BY MEANS OF THE GENERALIZED R-S-N-DIAGRAM

Miner has proposed as a measure M of the amount of cumulative fatigue damage

$$M = \Sigma \; n_i / N_i \qquad (5)$$

where

n_i = the actually imposed number of cycles of the i:th load and

N_i = the corresponding fatigue life

with the criterion that failure will occur when M reaches the value $M = 1$. No consideration is taken to the probability of this failure.

Let us now compute the values of M for the two programs above. From Figure 3 we obtain:

Program nr.1: $M = 2/4 + 10/16 = 1.125$

Program nr.2: $M = 8/16 + 1/4 = 0.75$

This result proves that Miner's measure is not independent of the order in which the different loads are applied, a conclusion which has been repeatedly verified by experiment, and which makes this rule very unreliable.

A more reliable result will be obtained by using the percentile r_p with properly specified value of P as a measure of the cumulative fatigue damage and with the criterion, that failure will occur with the

6

probability P%, when $r_p = S_{max}$.

6. ANALYTICAL REPRESENTATIONS OF THE r_p-FUNCTIONS

It is always possible to determine these functions by properly designed fatigue tests, but analytical representation of them are very desirable.

As a first approach the following expression is proposed, because it covers essential properties of the functions.

$$dR/dN = -a(S-S_o)^b/(R-S)^c \qquad (6)$$

where R stands for anyone of the percentiles r_p and a,b,c are coefficients, which will be experimentally determined. It is not excluded that these coefficients are independent of P.

7

7. REFERENCES

1. Broek, D.: "The Residual Strength of Aluminum Alloy Sheet Specimens Containing Fatigue Cracks or Saw Cuts", NLR-TR.M2143, Feb 1965.
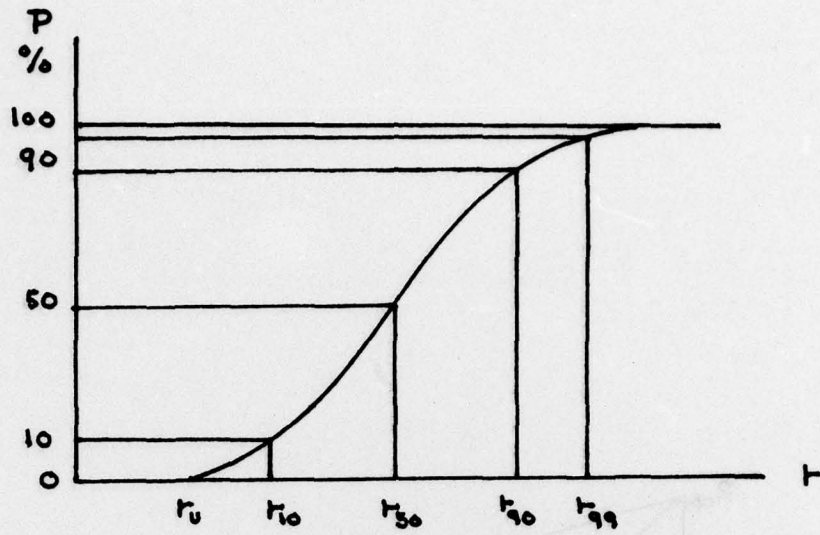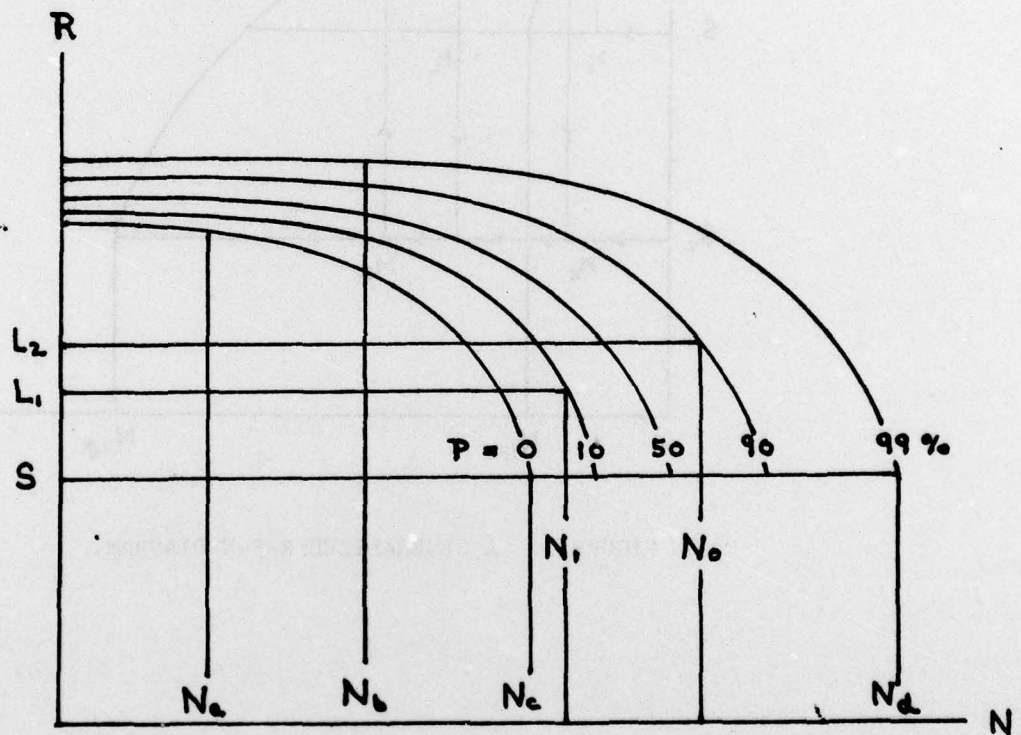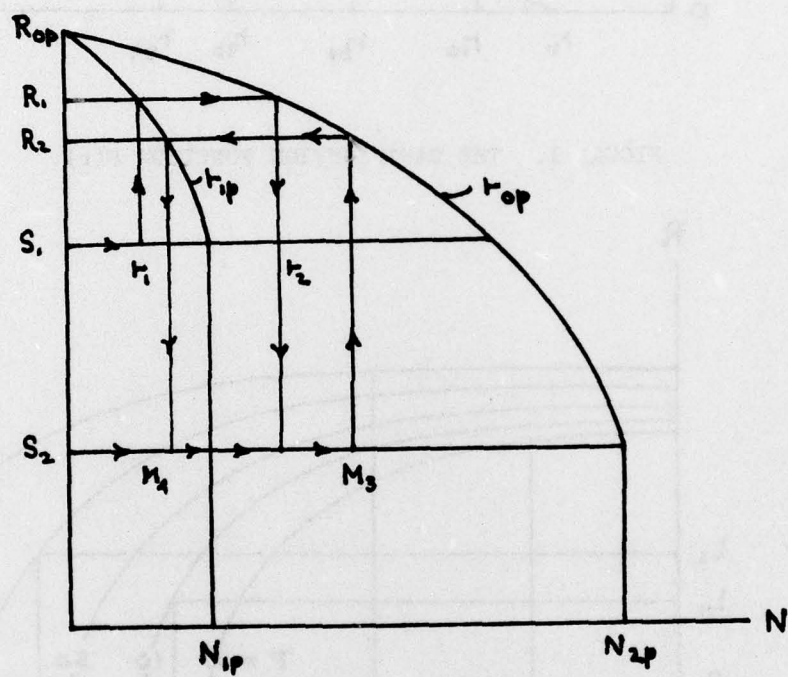
8

FIGURE 1.   THE DISTRIBUTION FUNCTION F(r).

FIGURE 2.   THE R-S-N DIAGRAM.

9

FIGURE 3.   A GENERALIZED R-S-N DIAGRAM.